A simple Bayesian linear excess relative risk model

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Radiation epidemiology

- A-bomb survivors
- Occupational studies (nuclear workers, radiologists)
- Radiation accidents
- Radiotherapy
- CT scan studies

Excess relative risk models

Concretely the linear relative rate model

$$\mathrm{R}(D) = \mathrm{e}^{\eta}(1 + \beta D),$$

where D is a variable exposure of interest, e^{η} is the background risk and β represents the excess relative risk (ERR) per unit of exposure.

Poisson non-linear models

Consider a cohort study in which incident cases of disease have been ascertained over a period of follow-up.

Poisson regression models consist of counts of cases by person-time as an offset of the risk, cross-classified by levels of explanatory variables, *i.e.*

$$C \sim \mathrm{Pois}\left(PY\,\mathrm{e}^{\eta}(1+\beta D)\right),$$

where *C* represents the number of cases and *PY* is the number of person-years of follow-up.

New Bayesian model

It is assumed the log-linear term is

$$\eta = \alpha_0 + \alpha_1 x,$$
$$\alpha_0, \alpha_1 \sim \mathcal{U}(-\infty, +\infty),$$

where *x* is an indicator variable (*e.g.* smoker), and α_0, α_1 and β are independent.

For simplicity

$$\beta \sim \mathcal{U}(0, +\infty),$$

but this prior is open to any distribution with positive support.

Posterior distribution

Let $X=\{C, PY, x\}$, applying the Bayes' theorem, the posterior of $\{\alpha_0, \alpha_1, \beta\}$ is

$$\mathrm{P}(\alpha_0, \alpha_1, \beta | X) \propto \frac{\prod_{i=1}^{n} (PY_i \mathrm{e}^{\alpha_1 x_i}(1+\beta D_i))^{C_i}}{\exp\left(\mathrm{e}^{\alpha_0} \sum_{i=1}^{n} PY_i \mathrm{e}^{\alpha_1 x_i}(1+\beta D_i) - T\alpha_0\right)},$$

where $T = \sum_{i=1}^{n} C_i$ is the total number of diseases in the follow-up.

Marginal posterior of the ERR

The goal here is to get the marginal posterior of the ERR, the posterior distribution of β

$$\mathrm{P}(\beta|X) \propto \int_{\mathbb{R}^2} \mathrm{P}(\alpha_0, \alpha_1, \beta|X)\mathrm{d}\alpha_0\mathrm{d}\alpha_1 \propto \left[\prod_{i=1}^{n}(1+\beta D_i)^{C_i}\right] \cdot \left[\sum_{i|x_i=0} PY_i(1+\beta D_i)\right]^{N-T} \cdot \left[\sum_{i|x_i=1} PY_i(1+\beta D_i)\right]^{-N},$$

where $N = \sum_{i|C_i=1} x_i$.

Example: CT scans in childhood

Pearce *et al.* 2012 analysed the risk of leukaemia in young patients who were first examined with CT in National Health Service centres in England, Wales, or Scotland in a 23 years retrospective cohort study.

There were 74 leukaemia diagnosis for 178604 patients, and a total of 1,720,984 person-years in this study.

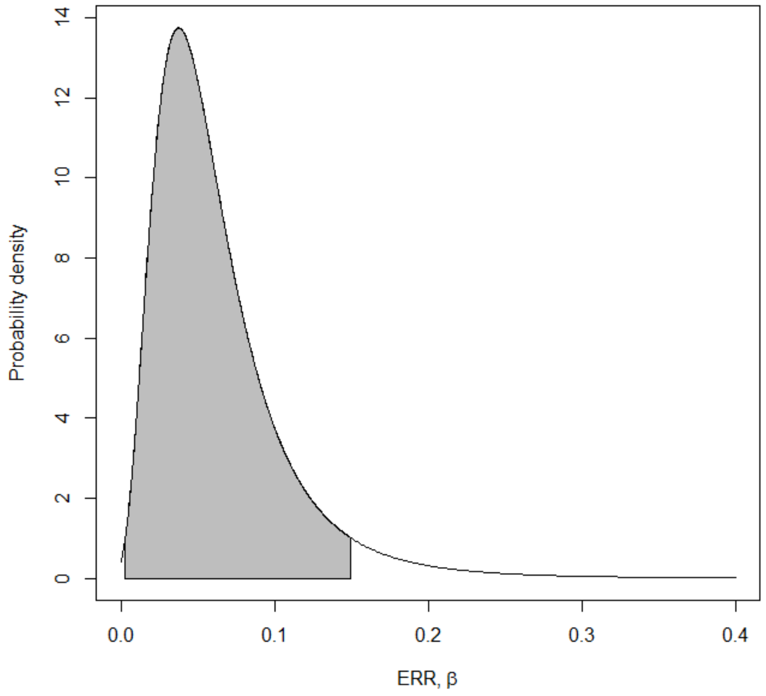
	Dose group (mGy)					
	<5	5-9	10-14	15-19	20-29	30+
Cases	15	17	12	11	4	15
Person-years	588450	438828	213289	244844	70523	165049
Mean dose per group (mGy)	2·32	7·08	12·34	16·54	24·69	51·13
Relative risk (95% CI)	1	1·44 (0·70, 2·99)	2·03 (0·89, 4·54)	1·53 (0·63, 3·59)	2·02 (0·56, 5·83)	3·18 (1·46, 6·94)

Results

Taking *x* as an indicator for patients whose attained age is >20, the posterior of the ERR (per mGy) results

- **Mode:** 0.038.
- **95% HPDI:** (0.003, 0.150).

In Pearce *et al.* 2012, using 5 categories of attained age, $\hat{\beta}$ = 0.036, 95% PLCI 0.005-0.120.



Conclusions and discussion

- The model presented here is simply and easy to implement.
- The Bayesian analysis provides an accurate framework for dealing with uncertainties, *e.g.* $P(\beta > 0.05) = 0.531$.
- To keep the closed form, the priors of both parameters at the background risk term are improper uniform priors.
- The prior choice for β can be changed easily, *e.g.* a gamma empirical Bayes using the Pearce *et al.* 2012 results, *e.g.* $\beta \sim \text{Gamma}(2.8, 50)$ results an ERR posterior with mode 0.037 and 95% HPDI (0.011, 0.091).

That’s all there is